RELIABILITY ASSESSMENTS OF REPAIRABLE SYSTEMS – IS MARKOV MODELLING CORRECT?

By K G L Simpson and M Kelly
(SILVERTECH SAFETY CONSULTANCY LTD)

1. EXECUTIVE SUMMARY

The suitability of applying MARKOV modelling techniques for redundant repairable systems has been questioned in a paper by Mr W G Gulland (Ref. 7.1). This paper further analyses the issue by considering a range of redundant systems and applying different techniques for calculating system failure rate and unavailability's. When comparing the results from the different techniques there is good agreement with the exception of the standard Markov modelling which shows a pessimistic difference of 2:1 for a 1oo2 system and up to 24:1 for a 1oo4 voted system. The paper then considers the reason for these difference and concludes that commonly implemented MARKOV modelling for repairable redundant systems, where the system is modelled with cascade repair to allow the steady state equation to be calculated, is not correctly applied because the process ceases to be a MARKOV process. An approach to compensate for the process not being a MARKOV process is then given to allow the correct solution to be obtained using the MARKOV mathematics. The paper finally concludes that there are better and simpler methods of analysing redundant safety system.

2. ANALYSIS OF A RANGE OF REDUNDANT REPAIRABLE SYSTEMS

In order to identify whether there is a potential problem with the MARKOV modelling technique the resulting formulae are compared with those derived using probability analysis and both sets of results are then compared numerically with those generated by applying Monte Carlo simulation. These three different approaches are used to analyse redundant systems, including both automatically detected faults and faults found by proof test. To keep the configuration of test cases to a reasonable number, each analysis is applied assuming either all faults are automatically detected or all faults are only found by proof test. The redundancy levels analysed are duplex, triplex and quadriplex. It should also be noted that in order to avoid the masking of differences in the results, common mode failures have not been modelled. The standard MARKOV modelling approach was then modified to correct for the differences found , see discussion following the tables.

The analytical methods compared were:

- Probability analysis of system failures — Appendix 1
- Simulation using Monte Carlo techniques — Appendix 2
- MARKOV Modelling, both standard and modified — Appendix 3

The results are summarised in the following tables:

Tables 1 and 2 compare the derived formulae for each redundant configuration:

TABLE 1: Comparison of Unavailability Formulae For Immediately Detected Faults

1 Configuration	2 PFD Using Probability Analysis (Appendix 1)	3 PFD Using MARKOV Standard Formula (Appendix 3)	4 PFD Using MARKOV Modified Formula (Appendix 3)	5 Ratio Factor For MARKOV Standard Results (column 3) to Correct Formulas (column 2)
1oo2	λ^2t^2	$2\lambda^2t^2$	λ^2t^2	2
2oo2	$2\lambda t$	$2\lambda\ t$	$2\lambda t$	1
1oo3	λ^3t^3	$6\lambda^3t^3$	λ^3t^3	6
2oo3	$3\lambda^2t^2$	$6\lambda^2t^2$	$3\lambda^2t^2$	2
3oo3	$3\lambda t$	$3\lambda t$	$3\lambda t$	1
1oo4	$\lambda^4\ t^4$	$24\lambda^4\ t^4$	$\lambda^4\ t^4$	24
2oo4	$4\lambda^3t^3$	$24\lambda^3t^3$	$4\lambda^3t^3$	6
3oo4	$6\lambda^2t^2$	$12\lambda^2t^2$	$6\lambda^2t^2$	2
4oo4	$4\lambda t$	$4\lambda t$	$4\lambda t$	1

These approximate formula apply when repair rate 1/t >>failure rate λ .

TABLE 2: Comparison of Unavailability Formulae For Faults Found at Proof Test Intervals

1 Configuration	2 PFD Using Probability Analysis (Appendix 1)	3 PFD Using MARKOV Standard Formula (Appendix 3)	4 PFD Using MARKOV Modified Formula (Appendix 3)	5 Ratio Factor For MARKOV Standard Results (column 3) to Correct Formulas (column 2)
1oo2	$\frac{\lambda^2 T^2}{3}$	$\frac{\lambda^2 T^2}{2}$	$\frac{\lambda^2 T^2}{3}$	1.5
2oo2	λT	λT	λT	1
1oo3	$\frac{\lambda^3 T^3}{4}$	$\frac{3\lambda^3 T^3}{4}$	$\frac{\lambda^3 T^3}{4}$	3
2oo3	$\lambda^2 T^2$	$\frac{3\lambda^2 T^2}{2}$	$\lambda^2 T^2$	1.5
3oo3	$\frac{3\lambda T}{2}$	$\frac{3\lambda T}{2}$	$\frac{3\lambda T}{2}$	1
1oo4	$\frac{\lambda^4 T^4}{5}$	$\frac{3\lambda^4 T^4}{2}$	$\frac{\lambda^4 T^4}{5}$	7.5
2oo4	$\lambda^3 T^3$	$3\lambda^3 T^3$	$\lambda^3 T^3$	3
3oo4	$2\lambda^2 T^2$	$3\lambda^2 T^2$	$2\lambda^2 T^2$	1.5
4oo4	$2\lambda T$	$2\lambda T$	$2\lambda T$	1

These approximate formula apply when repair rate 1/T >>failure rate λ .

Tables 3 and 4, gives comparisons of the numerical test cases to allow comparison of the results given in Tables 1 and 2 with the Monte Carlo simulation technique. Two test cases were considered, for detected faults each leg of the redundant system was assumed to have MTBF of 500 hr and a MDT of 15 hr and for faults found at proof test intervals each leg is assumed to have a MTBF of 50,000 hr and a proof test interval of 2500 hr. These figures were chosen to keep $\lambda t < 0.1$ and to keep run times on the simulation reasonable low.

TABLE 3: Immediately Detected Faults - Numerical Comparison

1 Configuration MTBF per element = 500 hr, MDT = 15hrs	2 PFD Using Probability Analysis	3 PFD Using Simulation	4 PFD Standard MARKOV	5 PFD Modified MARKOV
1oo2	0.9×10^{-3}	0.86×10^{-3}	1.8×10^{-3}	0.9×10^{-3}
2oo2	0.06	0.057	0.06	0.06
1oo3	0.27×10^{-4}	0.25×10^{-4}	1.62×10^{-4}	0.27×10^{-4}
2oo3	0.27×10^{-2}	0.25×10^{-2}	0.54×10^{-2}	0.27×10^{-2}
3oo3	0.09	0.085	0.09	0.09
1oo4	0.8×10^{-6}	0.85×10^{-6}	19.2×10^{-6}	0.8×10^{-6}
2oo4	0.11×10^{-3}	0.099×10^{-3}	0.66×10^{-3}	0.11×10^{-3}
3oo4	0.54×10^{-2}	0.49×10^{-2}	1.08×10^{-2}	0.54×10^{-2}
4oo4	0.12	0.11	0.12	0.12

TABLE 4 :Faults found by proof test interval - Numerical Comparison

1 Configuration MTBF per element = 50,000 hr, Proof Test Interval = 2500 hrs	2 PFD Using Probability Analysis	3 PFD Using Simulation	4 PFD Standard MARKOV	5 PFD Modified MARKOV
1oo2	0.83×10^{-3}	0.81×10^{-3}	1.25×10^{-3}	0.83×10^{-3}
2oo2	0.05	0.048	0.05	0.05
1oo3	0.31×10^{-4}	0.29×10^{-4}	0.93×10^{-4}	0.31×10^{-4}
2oo3	0.25×10^{-2}	0.23×10^{-2}	0.375x10-2	0.25x10-2
3oo3	0.075	0.072	0.075	0.075
1oo4	0.12×10^{-5}	0.10×10^{-5}	0.72×10^{-5}	0.12×10^{-5}
2oo4	0.12×10^{-3}	0.11×10^{-3}	0.36×10^{-3}	0.12×10^{-3}
3oo4	0.5×10^{-2}	0.46×10^{-2}	0.75×10^{-2}	0.5×10^{-2}
4oo4	0.1	0.094	0.1	0.1

Analysing the results for the faults found by proof tests (in tables 2 and 4) it can be seen that the formulas derived from probability analysis (Appendix 1) and the modified Markov (Appendix 2) agree and from the numerical test cases given in Table 4 these methods also agree with the Monte Carlo simulation results (Appendix 2). These results also agree with those given in the book by Dr D J Smith Reliability Maintainability and Risk chapter 8 (Ref 7.2) (it should be noted that for the proof test case the formulae given in that book were not derived using Markov) The normal Markov results (Appendix 3) are at variance with these results. The difference depends on the level of voting of the redundant elements, there being no disagreement when there is no voting up to a ratio of 7.5 with a voting difference of three.

Analysing the results for the immediately detected faults in Tables 1 and 3 it can be seen that the formulas derived from probability analysis (Appendix 1) and the modified Markov (Appendix 2) agree and from the numerical test cases given in Table 3 these methods also agree with the Monte Carlo simulation results (Appendix 2). The normal Markov results (Appendix 3) are at variance with these results The difference depends on the level of voting of the redundant elements, with no disagreement when there is no voting up to a ratio of 24 with a voting difference of three. The formula associated with immediately repairable faults given in the book by Dr D J Smith, Reliability Maintainability and Risk, chapter 8 (Ref. 7.2) have been derived using the standard Markov solution and therefore do not agree with the correct solution given in this paper.

The modified MARKOV technique for immediately detected faults relies on the premises that there will be negligible difference in the results if it is assumed that there is ‘N’ repair crew even if only one repair crew is used.

The Standard MARKOV analysis does not support this assumption. This is now discussed and analysed in the next section.

3. ONE REPAIR CREW VERSUS MULTIPLE REPAIR CREWS

Comparing the cases of single and multiple repair crews identifies the flaw in the MARKOV technique for repairable systems. Consider a simple illustration using MARKOV, as currently given in many textbooks, and as implemented in many commercial available MARKOV computer programmes. A 1oo2 redundant repairable safety system has a simple sub-system in each leg of the system. It should be noted that as a safety system it will only be required to function on detection of a potential hazard. Thus for safety analysis it is required to calculate the probability of the system working when demanded to. Applying MARKOV to this system yields the following results for Probability of Failure on Demand (PFD):

For 1 repair crew PFD = $2\lambda^2 (MDT)^2$

Whereas for N crews PFD = $\lambda^2 (MDT)^2$

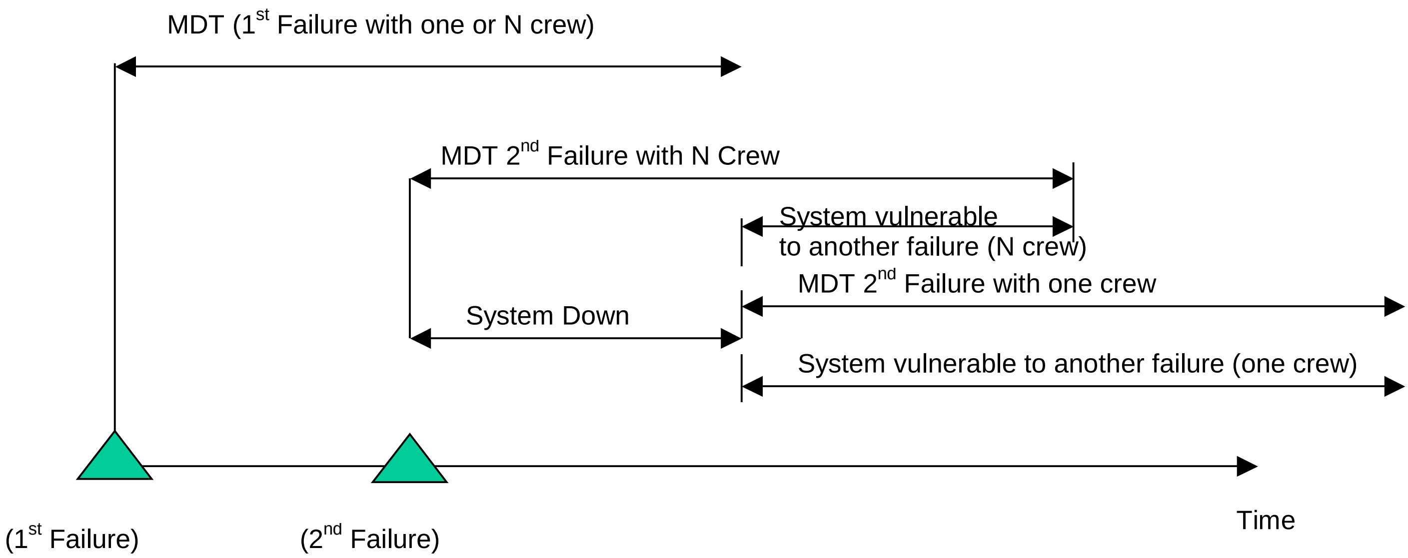
Thus MARKOV analysis indicates a 2:1 improvement by using multiple repair crews over when using one repair crew.

But consider how, in practise, the effect, on PFD, of having multiple repair crews over one repair crew.

Whilst there is only one fault at any one time in the system, having multiple repair crews cannot affect the results. The failure rate for a single fault $\lambda_F = 2\lambda$. Hence multiple repair crews can only affect the results when there are coincidence failures. Consider the case when there is a fault in one element of the redundant system, the overall system it still capable of performing its function since it is a 1oo2 voting system. The first repair crew start repairing this fault. Before this fault is repaired a fault occurs in the remaining leg of the system, and the system will now become unavailable.

Consider when the system will become available again, with or without multiple repair crews. Obviously the first repair crew will carry on the repair of the first fault and once this is repaired, the ‘system’ can be brought back on line irrespective of whether the second fault has been repaired by a second repair crew.

Thus the only impact of having a second crew is the length of time the second element of the system is down after the repair of the first fault i.e. the one repair crew system will be vulnerable to a second failure for the full repair time (MDT) whilst on average, with two repair teams it can be assumed that the second repair started half way through the first repair period. Therefore the exposure of the system due to this second failure is ½ MDT, as shown in the following diagram.



Hence, additional system failures, i.e. occurrence of two coincident failures, due to having one repair crew will be as the result of extending the repair time for second co-incident failures an average of 1.5 times normal MDT but will only apply to the fraction of failures that occur coincidentally and just after the repair of the first failure i.e. on a ratio of $< \frac{\lambda^2 t}{\lambda} \; MDT_1$

Thus the effect of extending one crew to multiple crews can only extend the average system MDT by a tiny fraction $\left(1 + 0.5 \times \frac{\lambda^2 \; MDT_1}{\lambda} \right)$ and does not reflect the 2:1 change given by traditional MARKOV analysis.
The error being in applying MARKOV for 1 repair crew, which is not a MARKOV process as discussed in the next section. However, the above demonstrates that the results obtained using N repair crews (which is a MARKOV process) can be used as for one repair crew. This cannot be applied for systems with faults found as a result of proof test intervals, in this case the repair transition rate has to be independently assessed. See Appendix 3.

4. CONCLUSIONS

The use of the Markov technique for modelling redundant repairable systems with automatic fault detection and one repair crew is flawed. This is because repair of multiple failures is not a Markov Process (namely that the probability of being in a state can be determined solely from knowledge of the previous state.)

For a redundant repairable system without a dedicated repair crew per equipment the transition from a multiple failure state does not depend on the repair of the last failure (as it should for the process to be Markov) but on the continued repair of the previous failure. For this reason a Markov model of this system is pessimistic as it underestimates the transition rate from the failed state. It is as if the repair crew abandon the earlier repair to carry out the repair of the latest failure.

With a dedicated repair crew per equipment the repair of the last failure is independent of preceding failures and the process is Markov and the calculations give the correct answer.

For a redundant repairable system with faults detected at periodic inspection for failed items the process is not Markov as the transition rate from the failed state (multiple failures) is a function of the time spent in the previous state (only one item failed).

However the Markov process can be adapted to provide correct answers since;

- for redundant systems with automatic fault detection it has been shown in this paper that an almost identical level of availability could be achieved with multi repair crews compared with a single repair crew (see previous section), and;

- for faults found at periodical inspection independently assessing the transition rate from the failed state and using that in the model.

This paper agrees with the basic issue given in the paper by W G Gulland (Ref. 7.1), namely that the standard Markov solution for redundant repairable system is incorrect. However the authors offer a different explanation for this error and have considered both immediately repaired faults along with faults found at proof test at intervals.

5. RECOMENDATIONS

From the analysis in this paper, it can be seen that the use of MARKOV modelling for redundant repairable systems has not in general been correctly applied, and one must question the use of a technique that can be widely used incorrectly. The main problem with this technique is that it is normal to model the complete system then apply a computer programme to produce the final answer. Hence, the user has very little insight into the make up and main contributors to the final answer and hence has limited ability to perform sensitivity checks to the solution.

Appropriate simulation techniques can be used instead of MARKOV with equal ease and do provide reliable results, however, similar to MARKOV there are the same serious disadvantage, as described in the previous paragraph, namely only one final answer is obtained.

One of the main reasons for performing hardware reliability analysis should be to allow the designer to assess the relative strengths of the various components that go towards making the complete system. The designer can then put his efforts in improving the weak links and simplifying the complex links. This is important because hardware failures are generally not the largest contributor to the systems overall performance and in many cases, the more complex links can introduce higher failures due to systematic and common mode, which can contribute a higher percentage of failures compares to the random hardware failures.

Thus the more useful method of analysis for these types of systems is to use reliability block diagrams or fault trees. This allows the designer not only to straightforwardly calculate the system reliability / availability but also to see the relative importance of each element.

The redundant equation can be derived using the same approach as given in Appendix 1. Contrary to most text books, theses and learned papers on this subject this method can easily cope with parallel elements with different failure rate, elements with multiple and different repair times and elements with both automatic detected faults / faults found by proof test etc. Only simple ratio / proportional maths is required to deal with these varieties.

6. ABBEVIATIONS

PFD	-	Probability of Failure on Demand
MDT	-	Mean Down Time
λ	-	Failure Rate
T	-	Time between Proof Tests
F	-	Element has Failed
OK	-	Element has not Failed
MTBF	-	Mean Time Between Failures
μ	-	Repair Rate
P	-	Probability

7. REFERENCES

1) W G Gulland, Repairable Redundant Systems and Markov Fallacy

2) Dr D J Smith, Reliability Maintainability and Risk (6th Addition Butterworth Heinemann ISBN0750651687)

3) William M Goble, Control Systems Safety Evaluation & Reliability,

4) F Lees, Loss Prevention in the Process Industries

5) AvSin+ program by Isograpth

APPENDIX 1
Probability Analysis of System Failures

Each system is analysed to ascertain the failure rate of every possible state, through every possible path that the overall system can be in. Then for any given voting configuration the states that are deemed as system failures can be added together to give the total system failure rate for the given failed conditions.

The following tables have a column to show the status of each leg of the redundant system plus a column for the overall system status. The leg status column shows whether that leg is still functioning correctly (OK) or has failed (F). The total system status column gives the system failure rate corresponding to the combined result of the leg statuses.

1) Assumptions:

$\lambda_{1, 2, 3, 4,}$	=	Individual legs of redundant systems failure rate.
λ_S	=	Total system failure rate.
$t_{1,2,3,4,}$	=	Mean Down Time.
T	=	Proof test interval.

If there is one failure in a given time 't' then on average it is assumed to occur half way in time period 't'. If there are two failures in a given time 't' then on average it is assumed that the failures occur equally spaced i.e. time between each failure is t/3. If there are three failures in a given time 't' then on average it is assumed that the failure occurs equally spaced i.e. time between each failure is t/4.

$\lambda t < 0.1$

$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda$

$t_1 = t_2 = t_3 = t_4 = t$

2) Consider Revealed Failures:

2.1 Duplex System

Leg 1 λ_1	**Leg 2** λ_2	**Total System** λ_S
OK	OK	1-Σ Failures
F	OK	λ_1
F	F	$\lambda_1 (\lambda_2 t_1)$
OK	F	λ_2
F	F	$\lambda_2 (\lambda_1 t_2)$

1oo2 $\quad \lambda_S = \lambda_1 (\lambda_2 t_1) + \lambda_2 (\lambda_1 t_2) = 2 \lambda^2 t$

$\therefore \quad PFD = \lambda_s t_s = 2 \lambda^2 t \times \frac{t}{2} = \lambda^2 t^2$

2oo2 $\quad \lambda_s \approx \lambda_1 + \lambda_2 = 2 \lambda$

$\therefore \quad PFD = \lambda_s t_s = 2 \lambda t$

The above analysis was repeated for triplex and quadplex systems and the results are given in Table 1

3) Consider Unrevealed Failures:

3.1 Duplex System

Leg 1 $\boldsymbol{\lambda_1}$	**Leg 2** $\boldsymbol{\lambda_2}$	**Total System** $\boldsymbol{\lambda_S}$
OK	OK	1-∑ Failures
F	OK	λ_1
F	F	$\lambda_1 (\lambda_2 T/2)$
OK	F	λ_2
F	F	$\lambda_2 (\lambda_1 T/2)$

1oo2 $\quad \lambda_S = \lambda_1 \lambda_2 T/2 + \lambda_2 \lambda_1 T/2 = \lambda^2 t$

$\therefore \quad PFD = \lambda_s T_s = \lambda^2 T\, T/3 = \underline{\dfrac{\lambda^2 T^2}{3}}$

2oo2 $\quad \lambda_s = \lambda_1 + \lambda_2 = 2\lambda$

$\therefore \quad PFD = \lambda_s T_s = 2\lambda T/2 = \underline{\lambda T}$

The above analysis was repeated for triplex and quadplex systems and the results are given in Table 2

APPENDIX 2

Simulation Using AvSim+ Program (Ref. 7.5)

The Monte Carlo program simulates the component failures in order to deduce the system reliability taking into account the redundancies, and maintenance / inspection intervals. This process involves synthesising system performance over a given number of simulation runs. Each simulation run in effect emulates how the system might perform in real life based on the input data provided by the user. The input data can be divided into two categories – a failure logic diagram and quantitative failure and maintenance parameters. The logic diagram informs the computer programme how component failures interact to cause system failures. The failure and maintenance parameters inform the programme how often components are likely to fail and how quickly they will be restored to service. By performing many simulation runs the computer programme can build up a statistical picture of the system performance by recording the results of each run.

Detected Faults

Configuration	Component M T B F (hr)	MDT (hr)	λ_S (1/hr)	PFD
1oo2	500	15	1.1×10^{-4}	0.86×10^{-3}
2oo2	500	15	3.8×10^{-3}	0.057
1oo3	500	15	4.9×10^{-6}	0.25×10^{-4}
2oo3	500	15	3.3×10^{-4}	0.25×10^{-2}
3oo3	500	15	5.5×10^{-3}	0.085
1oo4	500	15	2.0×10^{-7}	0.85×10^{-6}
2oo4	500	15	1.9×10^{-5}	0.099×10^{-3}
3oo4	500	15	6.4×10^{-4}	0.49×10^{-2}
4oo4	500	15	7.1×10^{-3}	0.11

TABLE A2-1

Simulation Results
Faults found at proof test interval

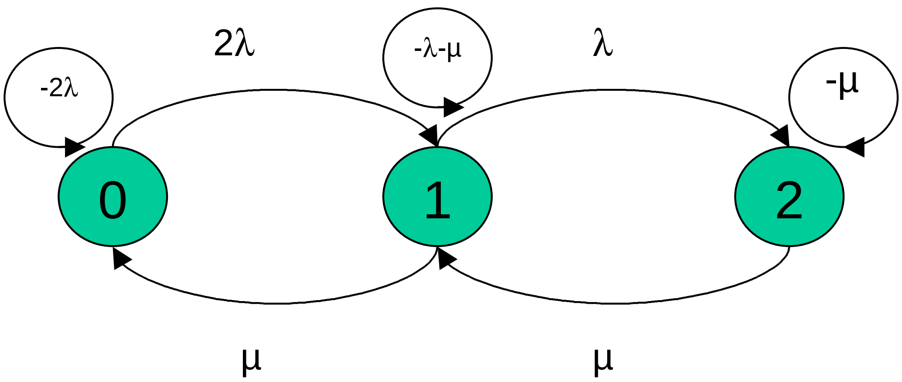
Configuration	Component M T B F (hr)	Proof test Interval (hr)	λ_S (1/hr)	PFD
1oo2	50,000	2,500	0.97×10^{-6}	0.81×10^{-3}
2oo2	50,000	2,500	3.8×10^{-5}	0.048
1oo3	50,000	2,500	4.6×10^{-8}	0.29×10^{-4}
2oo3	50,000	2,500	2.8×10^{-6}	0.23×10^{-2}
3oo3	50,000	2,500	5.6×10^{-5}	0.072
1oo4	50,000	2,500	2×10^{-9}	0.10×10^{-5}
2oo4	50,000	2,500	1.8×10^{-7}	1.1×10^{-4}
3oo4	50,000	2,500	5.4×10^{-6}	0.46×10^{-2}
4oo4	50,000	2,500	7.3×10^{-5}	0.094

TABLE A2-2

APPENDIX 3
MARKOV MODELLING

The same redundant systems are analysed in this Appendix using both the standard MARKOV Models and a modified MARKOV approach to compensate for the fact that the systems being analysed are not true MARKOV processes.

Standard Markov Models: Immediately Detected Faults
Markov Model for 2 units



It should be noted that the circular rate shown at each mode are commonly shown, and hence shown here for completeness, but are not actually needed to derive the equation which are simply the sum of the rate leaving plus rates entering each mode from other modes.

λ = Unit Failure Rate (1/MTBF)
μ = Unit Repair Rate (1/ MDT)

The equations for transitions between states are:

$$\frac{dP_0}{dt} = -2\lambda P_0 + \mu P_1$$

$$\frac{dP_1}{dt} = 2\lambda P_0 + (-\lambda - \mu) P_1 + \mu P_2$$

$$\frac{dP_2}{dt} = \lambda P_1 + (-\mu) P_2$$

The sum of the probabilities of states equals unity i.e.

$$P_0 + P_1 + P_2 = 1$$

At steady rate the transition equations become:

$$-2\lambda P_0 + \mu P_1 = 0$$

$$2\lambda P_0 + (-\lambda - \mu) P_1 + \mu P_2 = 0$$

$\lambda P_1 + (-\mu) P_2 = 0$

These equations can be solved by substitution:

$$P_1 = \frac{\mu P_2}{\lambda}$$

$$P_0 = \frac{\mu P_1}{2\lambda} = \frac{\mu^2}{2\lambda^2} = P_2$$

As $\mathbf{P_0 + P_1 + P_2 = 1}$

$$\frac{\mu^2 P_2}{2\lambda^2} + \frac{\mu}{\lambda} P_2 + P_2 = 1$$

$$P_2 = \frac{2\lambda^2}{\mu^2 + 2\lambda\mu + 2\lambda^2}.$$

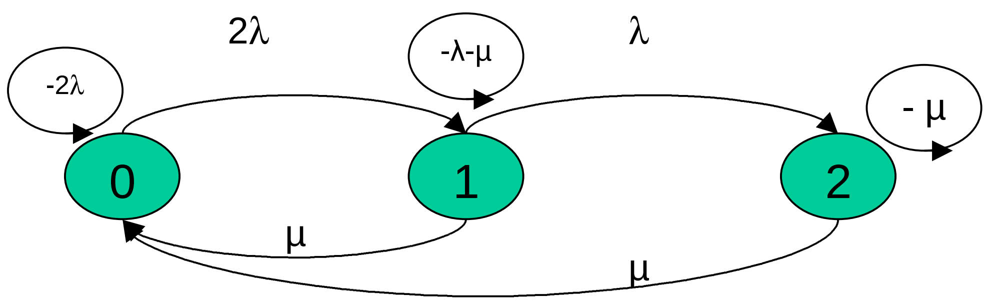
As $\mu >> \lambda$ Λ

$$P_2 = \frac{2\lambda^2}{\mu^2} = 2\lambda^2 t^2$$

Where t = average repair time = $1/\mu$ and P_1 (2oo2) is approximately $2\lambda t$

The above analysis was repeated for triplex and quadplex systems and the results are given in Table 1

Faults Found at Proof Test Interval
Markov Model for 2 units



λ = Unit failure rate (1/MTBF)

μ = Unit repair rate which is approximately 1/T/2 (where T = inspection interval) as the actual repair time is small in comparison to the inspection interval.

The equations for transitions between states are:

$$\frac{dP_0}{dt} = -2\lambda P_0 + \mu P_1 + \mu P_2$$

$$\frac{dP_1}{dt} = 2\lambda P_0 + (-\lambda - \mu) P_1$$

$$\frac{dP_2}{dt} = \lambda P_1 + (-\mu) P_2$$

The sum of the probabilities of states is units i.e.

$$P_0 + P_1 + P_2 = 1$$

For steady state the transition equations become:

$$-2\lambda P_0 + \mu P_1 + \mu P_2 = 0$$

$$2\lambda P_0 + (-\lambda - \mu) P_1 = 0$$

$$\lambda P_1 + (-\mu) P_2 = 0$$

These equations can be solved by substitution:

$$P_1 = \frac{\mu P_2}{\lambda}$$

$$P_0 = \frac{\mu(\lambda + \mu) P_2}{2\lambda^2}$$

As $P_0 + P_1 + P_2 = 1$

$$\frac{\mu(\lambda + \mu) P_2}{2\lambda^2} + \frac{\mu}{\lambda} P_2 + P_2 = 1$$

$$P_2 = \frac{2\lambda^2}{2\lambda^2 + 3\lambda\mu + \mu^2}.$$

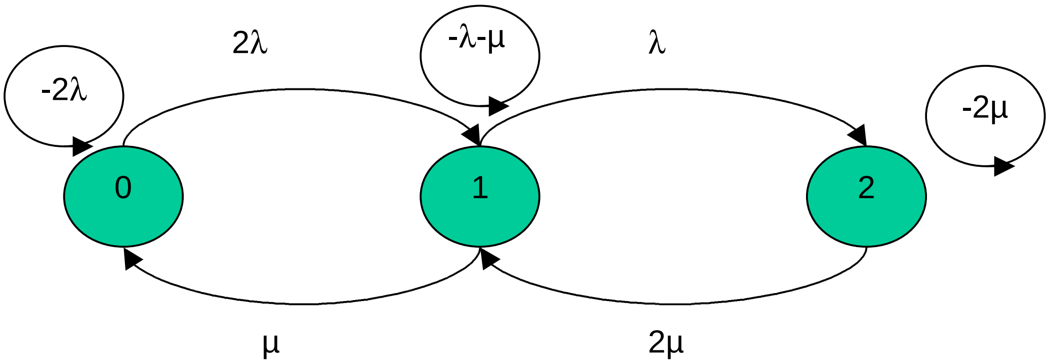
As $\mu >> \Lambda$ this approximates to

$$P^2 = \frac{\lambda^2 T^2}{2}$$

Where T is the inspection interval $= 2/\mu$ and P_1 (2oo2) is approximately λT

The above analysis was repeated for triplex and quadplex systems and the results are given in Table 2

Markov Model Assuming ‘N’ Repair Crew
Immediately Detected Faults
Model for 2 units



In this case it is recognised that the second failure will occur during the repair of the first unit. On average this will be half way so that the average time to transfer from State 2 to State 1 is half the average repair time t/2 = 2μ (where μ = 1/t) or assume ‘N’ repair crew.

The equations for transitions between states are:

$$\frac{dP_0}{dt} = -2\lambda P_0 + \mu P_1$$

$$\frac{dP_1}{dt} = 2\lambda P_0 + (-\lambda - \mu) P_1 + 2\mu P_2$$

$$\frac{dP_2}{dt} = \lambda P_1 + (-2\mu) P_2$$

Again the sum of state probabilities equals units i.e.

$$P_0 + P_1 + P_2 = 1$$

At steady state the transition equations become:

$$-2\lambda P_0 + \mu P_1 = 0$$

$$2\lambda P_0 + (-\lambda - \mu) P_1 + 2\mu P_2 = 0$$

$$\lambda P_1 + (-2\mu) P_2 = 0$$

These equations can be solved by substitution:

$$P_1 = \frac{2\mu}{\lambda} P_2$$

$$P_0 = \frac{\mu^2}{\lambda^2} P_2$$

As $P_0 + P_1 + P_2 = 1$

$$\frac{\mu^2}{\lambda^2} P_2 + \frac{2\mu}{\lambda} P_2 + P_2 = 1$$

$$P_2 = \frac{\lambda^2}{\lambda^2 + 2\lambda\mu + \mu^2}.$$

As $\mu >> \lambda$ this approximates to

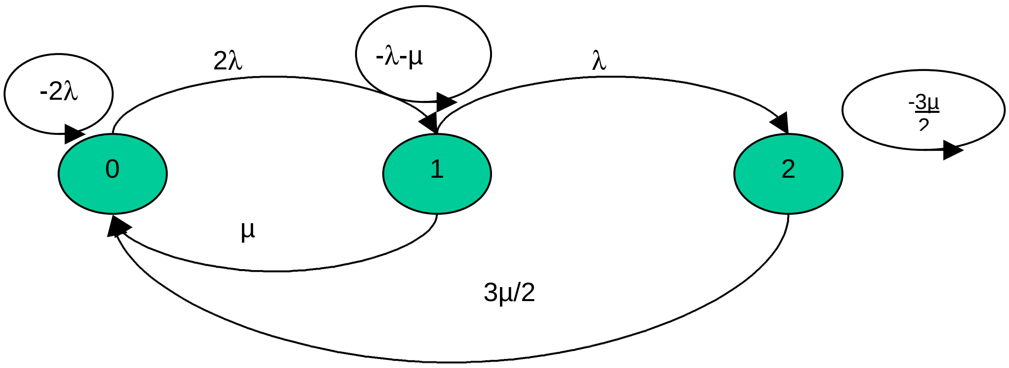
$$P^2 = \lambda^2 t^2$$

Where t = average repair time = $1/\mu$

P_1 (2oo2) = $2 \lambda t$

The above analysis was repeated for triplex and quadplex systems and the results are given in Table 1

Faults Found at Proof Test Interval with compensation for cascade failures
Markov Model for 2 units



In this case it is recognised that the second failure in the inspection interval will occur on average approximately 2/3 if the way through the interval. Therefore the transition rate form state 2 to state 0 if $3/2\mu$ since $\mu = 1/(T/2)$, T is the proof test interval i.e. $\frac{3}{T} = \frac{3\mu}{2}$

The equations for transitions between states are:

$$\frac{dP_0}{dt} = -2\lambda P_0 + \mu P_1 + \frac{3\mu}{2} P_2$$

$$\frac{dP_1}{dt} = 2\lambda P_0 + (-\lambda - \mu) P_1$$

$$\frac{dP_2}{dt} = \lambda P_1 + \left(\frac{-3\mu}{2}\right) P_2$$

The sum of all state probabilities equals unity i.e.

$$P_0 + P_1 + P_2 = 1$$

For steady state the transition equations become:

$$-2\lambda P_0 + \mu P_1 + \frac{3\mu}{2} P_2 = 0$$

$$2\lambda P_0 + (-\lambda - \mu) P_1 = 0$$

$$\lambda P_1 + \left(\frac{-3\mu}{2}\right) P_2 = 0$$

These equations can be solved by substitution:

$$P_1 = \frac{3\mu}{2\lambda} P_2$$

$$P_0 = \frac{3\mu(\lambda + \mu)}{4\lambda^2} P_2$$

As $P_0 + P_1 + P_2 = 1$

$$\frac{3\mu(\lambda + \mu)}{4\lambda^2} P_2 + \frac{3\mu}{2\lambda} P_2 + P_1 = 1$$

$$P_2 = \frac{4\lambda^2}{4\lambda^2 + 9\lambda\mu + 3\mu^2}.$$

As $\mu >> \lambda$ this approximates to

$$P^2 = \frac{\lambda^2 T^2}{3}$$

Where T = proof test interval $= 2/\mu$

P_1 (2oo2) = λT

The above analysis was repeated for triplex and quadplex systems and the results are given in Table 2